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ABSTRACT

This bulletin concerns the role of memorization in mathematics instruction. Sections of the bulletin are devoted to discussions of: old math vs. new math; the importance of memorization ability on mathematics learning; misconceptions about memory; how to enhance the memory, including short-term vs. long-term memory systems, attention, interest, visualization/representation, association/reconstruction, mnemonic functions in mathematics learning, characteristics of good and poor students as related to memory, an example of the application of these principles, and implications. No references are included. (MNS)

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MEMORIZING IS NOT A DIRTY WORD: WHAT SHOULD A MATHEMATICS TEACHER KNOW TO USE IT EFFECTIVELY IN TEACHING AND LEARNING MATHEMATICS?

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Mahesh C. Sharma

After the Russian Sputnik was launched, American educators began to seriously examine the American educational system. The most important outcome of this interest was the analysis of mathematics and science teaching methodology and curricula. It was discovered that the preparation of American students in mathematics and science learning was seriously deficient. The analysis showed that the major cause of this deficiency was that the mathematics and science curriculum was outdated and there was overemphasis on teaching mathematics and science using rote learning methods. It was found that the teachers relied heavily on memory techniques in teaching mathematics. As a result, learning by rote as a major method of learning and teaching mathematics and science was heavily attacked, and rightly so, by all segments of the then educational establishment. New and more comprehensive content along with new methods of teaching mathematics and science emerged on the American educational scene. It resulted in curriculum revisions and the era of the so called "New Math" began.

OLD MATH VS. NEW MATH

With the advent of this "New Math," a new approach to mathematics teaching and learning began. The structure and organization that is characteristic of higher mathematics was brought down to the elementary and ordinary secondary mathematics. This new organization in school mathematics placed a heavy emphasis on the deductive and logical reasoning in the conceptualization of mathematical ideas. Earlier this approach to the mathematics was generally reserved for selective high powered mathematics courses at the high school and college level. The core of this new approach to mathematics was to provide a logical and structural basis for every concept in order to facilitate understanding, which is an extremely important aspect of mathematics and its learning. But in many ways it was a clear departure from the previous methodology of teaching mathematics, where heavy importance was placed on presenting mathematical information through an intuitive approach and reliance on rote memory. The intiuitive approach, generally, is less rigorous. In the intuitive method of teaching there is heavy emphasis on the recognition of patterns in mathematical information and then extending those patterns. It required students to see the emerging patterns and relations of ideas and concepts in mathematics. It has many advantages but sometimes a student may acquire skills without adequate conceptual understanding. Sometimes students can even make inaccurate extensions of ideas. The other characteristics of the pre-New Math era was an undue emphasis on the algorithmic aspect of mathematics without the accompanying understanding of the development of that algorithm.

The most important impact of the new teaching approach and the new content was the perception in the minds of many teachers and the general public that rote memorization of formulas and mathematics facts did not result in any meaningful learning. Teachers began to consider teaching in the old way of relying on memory a sheer waste of their time and also a waste of time for the students. So the deemphasis on the use of memory in the learning process began.

IMPORTANCE OF MEMORIZATION ABILITY ON MATHEMATICS LEARNING

To learn is to remember. Learning is therefore dependent on memory. But memory has been given a short shrift of late in our teaching and learning methodology. We have put an inordinate amount of importance on the logical conceptualization process of mathematics. The word memory and memorization of facts and information became a dirty word not just in mathematics but in almost all subject areas. Have you seen any child memorizing poetry or important and wonderful pieces of literature? The primary philosophy in learning became that if one needed some information they should look it up. Open book examinations, providing all the formulas needed for answering questions in the examinations, and deriving the mathematical facts rather than committing them to memory became a norm rather than an oddity. In general, it is true that when you need some information if you can look it up there is really no need for memorization of the information nor is it always possible to do so. But this idea was carried so far that many students began to feel that it was not important to memorize any information at all. This was particularly so with the students who were not doing well in mathematics in the first place. Good students have always memorized inordinate amounts of information in order to be

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efficient and swift in what they do. The average and poor students were the ones who were most affected by this change in thinking about the use of memorization in learning.

As a result, the teaching methodology beginning with the early sixties and until the early seventies deemphasized memorization of facts and formulas. This meant that whenever a student needed a formula or a fact he could and should look it up. Towards the ends of the seventies, as a result, there were students in the ninth and tenth grades who did not know their multiplication facts and anytime there was a problem involving facts and formulas they would take an inordinate amount of time to complete these problems. They would have to digress from the main problem to subsidiary problems relating to facts in order to solve the main problem. These digressions would make the main problem difficult for them. It would take longer for them to solve any mathematical problem of some substance. This resulted in frustration and lack of motivation and interest in the subject. Students' performance in all areas of mathematical concepts went down. Lower scores could be seen in all types of mathematics examinations and tests, along with poor performance on all kinds of mathematical concepts – computations, problem solving, and other general applications of mathematics.

During the period of mid to late seventies the general public as a reaction to the sorry state of affairs of children's performance on mathematics tasks began to demand the reinstitution of pre-"New Math" teaching techniques. This, among other things, included a demand for "back to basics"; which meant different things to different people but in one form or the other placed a heavy emphasis on the computational aspects of mathematics. In many minds the meaning of the computational approach became, once again, an emphasis on the memorization of mathematics facts. Most textbooks produced during that period included and emphasized basic computation and many of the topics included in earlier so called "New Math" curricula were dropped. Ironically, once again, the scores, as indicated by several national examination projects and programs, in the computational aspects of mathematics are rising, and at the same time, the scores on the items relating to problem solving have either stayed the same or declined for some.

One can, therefore, make the following observation: that the methods used before the "New Math" era placed a heavy emphasis on the computational aspects of mathematics and that required an extensive use of those methods that rely on memorization techniques. It seemed to produce results as far as the mastery of basic skills were concerned. The techniques that were introduced during the "New Math" approach were definitely better in developing concepts and in understanding the interrelationships between concepts. Each approach has its flaws. At the same time there are definite strong points and advantages in using both approaches. In many ways, therefore, the problem is not the deficiencies in the approach and the methodology but in the implementation. If we analyze these two periods in the history of mathematics education, we find positive elements in both periods. And with some effort one can identify the positive elements from both approaches and the synthesis could serve as a basis for crystalizing the methodology that might be appropriate to teaching of mathematics in this era. The experiences of teaching using the two approaches can be utilized in articulating a comprehensive teaching methodology for mathematics.

The trouble is not with teaching mathematics in the "Old Math" way or in the "New Math" way but it is the dogmatic perception that one way is better than the other. What is needed is an analysis to answer questions such as: what aspects of mathematics require what kind of approach to teaching? It has been shown that different aspects of mathematics can be taught better using different approaches. For example, in certain aspects of mathematics learning and teaching, particularly when it comes to facts and formulas in elementary school and in definitions and axioms in higher mathematics we need an emphasis on memorization.

It is not the use of memorization in teaching and learning that is bad. What is definitely harmful is the undue emphasis that teachers used to place on rote memory and used it as the only means of acquiring mathematics concepts, facts and formulas. In the last two decades an enormous amount of research on human memory and its functioning has taken place which has not reached an average classroom teacher. In most cases teachers have very little information about how memory works and what are some of the best ways to take advantage of this phenomenally important function of human activity. In some cases, where information has reached the teacher, no training has been provided in utilizing this information properly. It should be an important task for teacher training institutions to think about ways to make optimal use of this information about the functioning of human memory network. The teacher training institutions should conduct extensive research into how the memory functions, how to make the optimal use of memory techniques in teaching and learning and ways to enhance the capacity and capability of one's memory. Once we begin to bring that research to the classroom teacher we will begin to see a revolution in the teaching and learning process. As most of the new research in this field has shown that if memory is properly used it can be a powerful tool in teaching and learning of any subject.

MISCONCEPTIONS ABOUT MEMORY

Many students somehow find it much more acceptable to say that they have difficulty in mathematics because they do not remember arithmetic facts and formulas. But the same students remember an enormous amount of information about sports, their favorite singers, and many other details without any difficulty. So their poor performance is not because of poor memory but due to the lack of its use in the specialized area of mathematics. The question is how can we use memorization techniques in learning and teaching mathematics.

What should a teacher do to invoke this potential of memory for learning and teaching mathematics? What can a teacher do to make sure that the student is able to realize the potential that s/he has as far as memory is concerned? Or is there a different kind of memory system involved when it comes to mathematics? If so how can it be developed and how can we utilize it for mathematics teaching?

What is Important for the Remembering Process?

When one does not remember something it could be because of three reasons. And here we are talking about normal situtations, that is, when there are no organic conditions that might be responsible for forgetting or losing the information once received by the person. *First*, the information might have not been received by the student in the first place either because of lack of

attention to the task, the information was not presented properly, presence of interference in learning that material, or competing demands for the student's attention. *Second*, the information presented to the student did not get transferred from short term memory to long term memory, too much information in a very short time, lack of appropriate rehearsal, or lack of anchoring of the new information. And *third*, the student may not know proper retrieval techniques, anxiety about the performance, past failures, or inappropriate methods used by the teacher to elicit response for retrieving the information.

Keeping this in mind, a good teacher has three important and crucial tasks: to make sure that the information entered the student's memory network, to make sure that he helps the student to transfer the information from short term memory to long term memory, and to employ proper retrieval methods. Each of these tasks requires special skills and technology of learning and teaching. That among other things involves understanding how the human memory works and what are some of the methods to improve it and exploit it for optimal learning.

HOW TO ENHANCE THE MEMORY

The important questions are: What is important for the remembering process? How is a new concept learned? Does a new concept replace, enhance, clarify or explain previous concepts? Do all these things happen simultaneously or one at a time? Do all of these functions happen because the concept is being transferred to memory? What happens to a concept that has been committed to memory? How long does it stay there?

At present, there are only partial answers to these questions. Scientists are just beginning to know about the functioning of human memory and its role in learning, although its importance has been appreciated as long as civilization has been in existence. All civilizations have developed methods of enhancing the memory systems. Some are based on observations and are useful others are based on simplistic understanding of the functioning of memory.

Learning means the capacity of a system to react in a new or a modified way as a result of experience. Every experience generates information. Memory is the capacity to store information that can later be retrieved with high distinction to steer the function which is correlated with new information. The new experience will not leave any traces if memory is not good. An experience becomes meaningful and useful because it gets connected with previously stored related information in the memory network. When the memory is better those connections are easy to make and therefore people with better memory are able to derive more from any particular experience. If a person has fewer things in the memory storage the possibility of making new connections is lower. Therefore when that person sees information (whether new or old) it looks as if it is all new. Since he can form only fewer connections as a result even the old information or previously taught topic looks newer, requiring greater effort to learn things. That is the reason that many students when they see even an old concept, give the impression that they had not seen that concept before. They do not have memory traces of that concept. They have not transferred the information from the short term memory to long term storage. If one has more information in their memory storage they are capable of receiving and storing more information because they can connect the incoming information with the information in their memory storage by establishing relationships and discerning patterns.

At the acquisition of memory, an external stimulus causes an internal transition in brain cells from one state to another by a small energy expenditure, and the new state of mechanism persists. At retrieval, the stored information is matched with information incoming from the environment.

Short Term vs Long Term Memory Systems

Short term and long-term memory, with an interposed period for fixation, are terms of special significance. What do we mean by long term and short term memory? Memorization essentially means the transfer of information from short term memory to long term memory, although in learning both are of great importance. The questions most important to answer from the point of view of educations are: How does one transfer the information from short term memory to long term memory or more importantly how can an idea from short term memory transfer to long term memory?

It is assumed that short term memory can be ascribed to transitory changes in synapses mainly of an "electrical nature." During the first hours after training, animals are highly sensitive to interference. But once a fixation of learning has occurred (10 seconds to several minutes), the stored information can withstand interference. Although, it could be shown that long term memory is not dependent on the maintenance of electrical activity circulating over certain neural nets.

A large amount of loss of information generally takes place immediately after receiving. It is therefore important to utilize the last five to ten minutes of the presentation/teaching to reviewing the material presented during that instructional period. This is the time when consolidation of presented information should take place. Long term memory has, therefore, generally been assumed to have its basis in long-lasting anatomical or molecular changes. This consolidation of learning can best take place by oral review: asking a lot of oral questions by breaking down the main task or concept into very small units. The whole algorithm can be broken down into its primary components by using effective task analysis techniques. For example, let us assume that the teacher has presented the concept of factoring trinomials.

Factor the trinomial:

$$x^2 + 7x + 12.$$

The factors of the trinomial are $(x + 3)(x + 4)$. After several examples, the teacher derives the general principal as:

$$x^2 + (a + b)x + ab = (x + a)(x + b).$$

In other words the factors are obtained by first identifying the factors of the constant term, then selecting those factors that add up to the coeficient of the middle term. As in the case of the above example the constant term is 12. All the possible factors of 12 are 1, 12; −1, −12; 2, 6; −2, −6; 3, 4; −3, −4. When we add all the pairs of factors we get 13, −13, 8, −8, 7, and −7. Out of all these the sum that appears in the middle term is 7 therefore we choose the pair as 3 and 4. And thus the factors found to be are $(x + 3)(x + 4)$. The teacher should solve several problems like this and then help students derive the general principle as outlined above. The review process should include questions as described below. We will take an example and treat it as if we are asking questions in a real classroom situation. The questions are asked orally and each student is asked at least 20 questions in a 10 minute period. (For any information or fact to be stabilized in the long term memory, one should see, hear, handle, or visualize from

18 to 25 times on the average when it is presented. Of course this number varies with an individual.)

The nature of the questions to be asked i.e. whether to ask difficult questions or easier ones should depend on the teacher's knowledge about the particular student. The questions should be progressively difficult and should cover the whole concept. The teacher should ask difficult and challenging questions when he is asking a question of a student whose cognitive and intellectual level is higher than others. The key to the questioning process is to do a thorough task analysis of the concept and algorithm under discussion. Here is an example of the questioning process when we want to find the solutions of the equation:

$$x^2 - 11x + 30 = 0.$$

Q. Is the expression on the left in the above equation a trinomial?
A. Yes.
Q. Give me an expression that is not a trinomial. (Ask several students to give such an example.)
Q. Is the trinomial in the equation above written in the standard form (in the descending powers of x)?
Q. What is the constant term in this equation?
Q. What is the coefficient of the middle term in this expression?
Q. What are all the factors of the number 30?
A. (One Student) 1, 30.
A. (Another Student) 2, 15.
A. (Another Student) 3, 10.
And so on . . .
Q. Any other set of factors?
A. –1, –30.
A. –2, –15.
And so on . . .
Q. What is the sum of 1, 30?
Q. What is the sum of –1, –30?
And so on . . .
Q. Which of these sums is the same as the coefficient of the middle term?
A. –11.
Q. Which factors gave us the sum –11?
Q. Which numbers will give us the factors?
A. –5 and –6.
Q. What are the factors of the trinomial in the equation?
A. (x – 5) (x – 6).
Q. What are the solutions of the equation?
A. x = 5 and x = 6.

Repeat this process with dozens of trinomials with all possible variations in the trinomials. This is a very powerful means of reinforcing the information that a student has just learned, to send the newly acquired information from short term memory to long term memory, and to involve all the students in the whole learning process. The same process should be repeated at the beginning of teaching a new concept. Before you begin the topic you plan to develop you should review the primary components of the topic through the oral review process.Five minutes of the oral review process at the beginning and the end of the lesson is enough. Although in my personal teaching whether I am teaching elementary children or a calculus class, an individual or a whole class, I devote more than fifty percent of my time in the oral questioning process. There has to be a certain amount of caution to be followed. First, some students due to their previous failures and threatening experiences in mathematics freeze when you ask them oral questions in front of the whole class. In that situation you should begin with very simple questions. Second, you should ask such a question so that the student is sure to answer the question. In case he does not or could not answer the question then you should back track and ask a simple question and keep on increasing the complexity of the question till you reach the question you asked in the first place. Third, the seating arrangement has to be in a semi-circle format. The traditional format of desk-table, desk-table, etc. is not conducive to this kind of teaching. You must be able to see each student in the eye when you are asking a question. You have to be on your toes. You have to know the answer the instant you ask the question. If you cannot answer the question instantly the process will not work.

Not much is likely to get into long term memory unless one attends to the task to be learned. Therefore the teacher or the parent should not solve the problem for the student, the child should solve the problem himself if necessary under the teacher's guidance. This has at least three advantages. First, when the student is solving the problem, he has to focus on all the aspects of the task and therefore his attention is guaranteed. Second, by using his senses particularly in the process of writing the mathematical terms and equations at least the psychomotor memory is being activated. Third, when the student is solving the problem he is also activating thinking and connecting skills along with the visual input to his memory network. Do not do every aspect of the work for the child, give him the directions to do it. He should do it in the first place. You need his attention. If you solve the problem for him, there is no guarantee that you had his attention in the first place. You cannot talk of foregetting something if the child never received or understood the idea in the first place. You have to bring the task to the automatic level, a few units of the task at a time. Failure to pay attention is a common reason for "forgetting." Failure to pay attention is a reason also for absentmindedness. The secret of successful memorization hinges on the ability to organize information as it is filed away in one's memory network.

Better memorization is dependent on the following principles of learning: attention, interest, meaningfulness, association, organization, and visualization.

Attention

Learning and remembering are dependent on coding. Novelty, information processing, and item storage and retrieval are all secondary to the presentation of the information by the teacher. The methods that the teacher uses in presenting the information determine how the student is going to process it and then utilize it. It is important to realize that all of us process information differently. It should therefore be apparent to every teacher that for optimal reception of the information it should be presented in multi-mode: visual, auditory, and tactile (kinesthetic). This ensures the reception of the information by the student for processing. And nothing can enter the memory system if it has not been received. When a student says that he does not remember certain information one reason might be that he did not receive the information in the first place. So the reception of the information is the first thing in the learning process.

The context, the set of events in which information occurs and is received by the learner is an important determinant of the outcome of receiving that information and eventually the ability to recall it. Many experiments dealing with the functioning of memory have shown that the information-carrying signal itself is dependent on the context. And yet, it is usually the context of learning that remains unspecified, unstudied and often hopelessly shrugged off as being

impossible to pay attention to both in its application to the use of memory and as a research subject. When the information presented to the student is connected with something he already knows, possesses, and has mastered then the processing of the new information is optimal. Whenever one encounters new information one evaluates it in the light of previous experiences if the new information validates, complements, connects or contrasts the previously acquired information it results in positive evaluation. It is accepted. Acceptance means that person is willing to store it; learn it. Therefore every new concept to be taught by the teacher must be presented in such a way that it relates and connects with what the student already knows. Fragmentation of information in terms of units, modules, and lessons where they are compartmented and remain unrelated with each other may not result in optimal learning. Connecting the new information with "old" information sets a strong context for the "new" learning. The conscious effort in connecting the new information with old information, skills, knowledge also helps the teacher to invoke a high degree of motivation on the part of the student.

Another important consideration in setting up the context is to help students to discern the emerging pattern in the incoming information and planning for the extension of that emerging and recognized pattern. By identifying the pattern the student begins to anticipate the incoming information and therefore is ready to receive and process the information.

Interest

We remember things that interest us or concern us. It is therefore very important that a teacher interests the students in the subject matter. The mathematics word problems he plans should involve topics that might have some intrinsic value for the student. The success that a student experiences works as an important motivator. Every lesson should include some activity that can provide some successful experience for every student in the classroom.

Visualization/Representation: A spatial encoding process

There are two important memory mechanisms: Spatial and temporal. Each relates to specific aspects of the incoming information and in the manner in which that information is processed, coded, remembered, and utilized.

Two processes dear to image makers are identification and imitation. In our recent endeavors of teaching, emphasizing only the sequential and temporal, are we are neglecting the powerful processes of image making for the purposes of learning and educating? Learning through reinforcement, particularly visual reinforcement, is extremely potent. We should not lose sight of the picturesque, for the brain is built to work with pictures as well. The perceptual mechanism is constituted in such a way that neural holograms are produced and that images can be reconstructed from the interaction of the holographic patterns.

What is the relevance to education of a spatially encoded neural representation? Could it be that the academic community has centered too much on the teaching of skills and habits using deductive and formal reasoning to the neglect of image? Is there something to be gained from the techniques developed by Madison Avenue? If indeed image formation is an important part of the memory mechanism, which it is, should we not listen to these experts, the image makers? We should utilize the techniques developed in other fields to maximize the use of visual techniques in teaching and learning of mathematics. In the early sixties some of the curriculum development centers and National Educational Labs developed some excellent films relating to mathematics at the college level which were very good in developing conceptual ideas in mathematics. The same kinds of films and video films can be developed and utilized in teaching of mathematics in schools. This kind of visual reinforcement takes place in almost every other area of learning so why cannot such an approach be utilized in mathematics to reinforce the learning through visual and concrete manipulations.

In mathematics learning a heavy use of imagery is made. Those students who have ability in "imaging" do better in mathematics. This ability can be developed. At the same time the use of imagery can be prevented either by presenting material very rapidly when a heavy emphasis is placed on the sequential deductive aspects of algorithms or, when an interference is present due to lack of attention or other factors. Therefore sequential and deductive presentation of mathematics material should be strongly supplemented by visual inputs and pattern recognition.

Association/Reconstruction

Events remembered vary according to the set and setting. An hour's conversation about childhood experiences would result in stimulating memories dating to childhood experiences; a discussion of an athletic event would result in the stimulation of memories associated with that event and related associations. The remembering process must operate by assembling from their distributed location the variety of fragments dismembered during storage. Apparently, the qualitative differences are determined by the set and setting of the moment and its immediate antecedent, not by the reconstructive process per se. We have all made this observation. Events dimly remembered become vivid, when we return to the scene of the experience. Meeting old friends, hearing familiar music, rereading a long-neglected topic, all call forth reconstruction of thought long since gone. We are only a little aware of the amount of remembering that is carried "out there" – not in our brain but in our homes, jobs, and libraries. Given these highly structured inputs, the machinery of our brain can restructure – reconstruct a rememberance from the bits and dabs actually stored in the head. The process is similar to that familiar to us in a desk calculator, less familiarly, a computer. Given the appropriate input, machines can perform many operations with the help of all the operations stored in their brains. A human being is much more than any machine and therefore is capable of utilizing previously stored information in the brain to learn more and create more.

The key factor in reconstructing any experience and therefore memory of an experience and the related information is reflective thinking. When a person reflects on an experience the related information can be reconstructed. This can be done in three ways: by guided questioning, retouching the memory traces by reflective thinking on a regular basis and providing students skeletons of the experiences.

Memory is not a single recording; one is able to remember by drawing up many different mechanisms of the brain, some of which we actuate consciously, while others are activated for us subconsciously. The more associations a person makes between the incoming information and the pre-existing knowledge, skills and information the better the "anchoring" of the new learning is.

How can we make our students better information-processing mechanisms? Here we are not talking about our students in terms of some kind of inhumane form by using the term "mechanism." We believe that in terms of information processing, human beings have the same characteristics as a super-computer that has an inordinate amount of capabilities. The lack in manifestation of these capabilities, as we see, is the learner's lack of understanding and mastery in memory techniques and in using these capabilities. There are people who remember incredible amounts of information. For example, they can commit to memory instantly the product of 37634 x 349289 and for that matter any combination or form of numbers. Yet none of these facts is stored in the machine. The operation of multiplication is reconstructive. It appears likely that a great deal of what we call remembering is reconstruction.

Mnemonic Functions in Mathematics Learning

The use of mnemonic devices in either remembering formulas or facts is an important strategy. Almost all teachers have used mnemonic devices to help children in remembering often used facts and operations. Everybody remembers from their school experience the popular mnemonic devices that their teachers used.

- **PEMDAS** (**P**arentheses, **E**xponents, **M**ultiplication and **D**ivision, and **A**ddition and **S**ubtraction) to remember the order of arithmetic operations.
- **K**ids **H**ave **D**ropped **O**ver **D**ead **C**onverting **M**etrics (**K**ilometer, **H**ectometer, **D**ekameter, **M**eter, **D**ecimeter, **C**entimeter, **M**illimeter), is a mnemonic device to remember the prefixes for the metric units.
- In converting percents to decimals and vice-a-versa we use the following device: When converting percents to decimals move the decimal two places to the left (we are going from P to D) and in converting decimals to percents move the decimal two places ahead (we are going from D to P). (In the English language alphabets the letter D comes before the letter P. So from percent to decimal P goes back to D and from decimal to percent D goes to P, ahead.) The location of D and P in the alphabet is serving as a mnemonic device.

There are many other examples that individual teachers have devised to help their students remember specific aspects of concepts and procedures. It is not just in mathematics that these devices are popular, in physiology to remember parts of anatomy and operations of organs students use all kinds of mnemonic devices.

Characteristics of Good and Poor Students as Related to Memory

The memorization ability of able and poor students are characteristically different, particularly in ways that they use in in the learning mathematics processes and concepts. For example: Superfluous, unnecessary data (if they have been introduced in a problem) are usually not remembered by capable students. Most of them forget a problem's content soon after solving it, but have a good memory for the method of solving problems of a certain type – the genuine structure of the operation.

The average pupils, on the other hand, remember concrete data and numbers relatively well, but they remember a problem's type characteristics less well or do not recall them at all. We find that average students try equally hard to remember the general and the particular, the abstract and the concrete, the essential and the nonessential. It can be said that their memory is "overburdened with superfluous information."

The capable pupils, on the other hand, retain the problem type and the generalized scheme of solving it. They pay less attention to the specific aspects of the problem. Therefore, when they see another problem of the same type they try to recall the type and the pattern of the problems that they have already solved rather than comparing the specific similarities between the problems.

The essence of a mathematical memory, therefore, consists in the generalized recollection of typical schemas of reasoning and of operations. As far as memory for specific data, for numerical parameters, it is "neutral" with respect to mathematical ability. Whether it is a strength or weakness, it does not in itself determine ability or inability in mathematics. The success in mathematics is least of all based on the mechanical remembering of a large number of facts, digits, numbers, formulas, and the like. It is useful, of course, to have such a memory, but most mathematicians do not have remarkable memories. They have memory for generalizations. They remember patterns and therefore are able to discern and recognize patterns when they encounter them. Many of the great mathematicians have also indicated that an extraordinary memory is by no means obligatory for a mathematician. Their writings emphasize the point that there is no need for a mathematician to preserve the entire proof of a theorem in his memory. He need remember only the initial and the final point and the idea of the proof. They emphasize the relationships between ideas.

The mathematical memory of capable students is generalized and operative related to the retention and the possible rapid effectation of generalized relationships, in the realm of number and letter symbols. In other words, the memory of a mathematically able student is markedly selective: the brain retains not only the mathematical information that enters it, but primarily that which is a "refind" of concrete data and which represents generalized and concrete structures. They finally remember the abstractions from concrete experiences of specifics not necessarily the specifics themselves.

This is the most convenient and economical method of retaining mathematical information. Retaining information in generalized and abbreviated form (in "condensed" forms) does not overload the brain with surplus information and thus permits it to be retained longer and used more easily.

It is not a matter of incapable students having poor memories in general and even of their having poor memories for generalizations. Many of them make satisfactory, even excellent progress in their other subjects. Many of these pupils exhibit good memories in nonmathematical areas such as: literature, geography, biology, history, with a good recall not only for factual, concrete material but also for ideas, patterns of reasoning, generalizations, and conclusions. Thus a pupil's memory may be good in general and its insufficiency may be detected only in the realm of mathematical operations, relations, and symbols.

An Example of the Application of These Principles:

At the Center for Teaching/Learning of Mathematics memorizing is not a dirty word. We want children to succeed in mathematics. We want them to learn mathematics with understanding and facility. We strongly believe that memory plays a very important role in learning anything. We want children to memorize mathematical facts, operations and other relevant

information, but it is done by keeping several principles in mind. Just asking an individual to memorize certain information is not enough. We have to provide techniques and methodology that can be useful for them to achieve the goal of committing a large amount of information to memory. As a matter of fact we have found that with appropriate techniques one can remember a large amount of information without "overloading" or any of the other myths that perpetrate the field of education. Here are some observations which we have abstracted from our experiences with children and adults who have taken advantages of our services at the Center. We find these observations useful in our work:

(a) We have observed that children are able to commit something to memory easier when it has been presented through a multi-modal approach. Mathematical concepts are presented to them through all the three modes – visual, auditory, and kinesthetic.

(b) It is easier to memorize a piece of information when the information presented has some valuable meaning attached to it, therefore every care is taken to attach meaning to the concept presented. We try to connect the new information with the old concepts that they already possess and understand.

(c) Every lesson begins with the reveiw of the material from the previous session and it ends with the reveiw of the material covered in that session. Before a new "secondary" concept is introduced all the related "primary" concepts are reviewed. (Primary concepts are those concepts that are independent, self standing, autonomous simple ideas. Secondary concepts on the other hand are those concepts that are dependent on two or more primary concepts.)

(d) The information to be memorized is organized in pieces and chains or "chunks of pieces" so that rehearsal activities will help the child to make connections. There are three important instances when the reinforcement or review is most useful. (1) Reviewing the primary concepts relating to the concept to be presented. (2) Reviewing the concepts at the end of the presentation. (3) Reinforcing learning through properly planned home-work assignments. The home-work assignments consist of three components: One-third of the assignment should include problems exactly similar to the problems covered during the lesson; another third should include cumulative review – problems relating to all the previous work; the final third includes problems that are included with partial solutions and with detailed directions.The third segment of the home work is to give a preview of the concepts to be learned and applications of the concept that are finished. But the key is to give a partial method and directions for the application. In the case of those children whose parents are intimately involved in child's learning process there is a fourth component that involves the parent asking oral questions relating to the material covered so far.

(e) We have observed that it is easier to memorize any information if one can discern emerging patterns in the information and can make connections with other known information. Therefore we make a conscious effort to help children in identifying the patterns and then to extend them. Various ideas, methods, and processes introduced are always connected with what student already knows or has mastered. The succeeding operation and concept are always connected with the preceding operations and every concept is always introduced as an extension of the previous one. Every care is taken to avoid fragmentation and compartmentation of ideas, procedures, and concepts. For example, if we want to introduce the product of algebraic expressions (a + b) and (c + d) for the first time then we will remind the student of the multiplication of two two-digit numbers:

85
x 39

and explain the sequence of activities in the operation, i.e., first we multiply 9 with 5, then 9 with 8, then 3 with 5 and write the product in the hundredth's place, and finally we add the two rows of numbers. After doing this we copy the process for multiplying

a + b
x (c + d)
ad + bd
ac + bc
ac + ad + bc + bd

The most important thing here is to emphasize the similarities and differences in the arithmetical and the algebraic operations and to make real connections between the two. We have found this kind of "connection making" helps students not only to understand the concept better but also to memorize the necessary information easier. Of course, after the concept has been mastered, we will introduce the method of finding the same product by using the popular process known as "FOIL" – first we multiply the "firsts" in the expressions (a and c), then we multiply "outers" in the expressions (a and d), then we multiply "inners" in the expressions (b and c), and finally we multiply "lasts" in the expressions (b and d), and then we combine like terms.

Implications

For education the moral is clear. Instruction (shared discovery of structure) should supplement teaching (showing). The tools for structuring and restructuring must be developed by the pupil, of course by the help of the teacher; the machinery of reconstruction must be put together. The techniques of analysis and synthesis are to be emphasized. The simple repetition of loosely connected facts ought to give way to search for structure in the material to which the student is exposed. The short answer test, which explores the number of items retained (ever so briefly and meaningfully), ought to be recognized for what it is – a labor saving device that tests only a small component of mastered material. It is important, but it does not test the mastery at all levels: intuitive, concrete, representational, abstract, applications, and communications.

As an educational experience, oral tests, term papers, projects, take home examinations, group tests, actual problem formulations and solving of those problems constitute learning. It is the combination of all of these efforts that produces a lasting effect as a learned activity. How many of you still have in your files somwhere a paper you wrote in high school or college of

which you were so proud, and still are? How many of you had the experience of reading a statement by Toyenbee, or other expression of some other famous person in the news media and exclaiming: Why, you know, I said something like that in a paper I wrote in that history course back in 19xx? The writing of the paper was educational, and today's reconstructive experience is possible and meaningful only because the paper had been written. The likelihood is slight that any one of the 500,000 short answer questions you have had to answer during your school life had much effect other than the deleterious one of dulling interest.

The understanding achieved by research on brain functions suggests that the present educational efforts is deficient in the techniques of image making and in the lack of emphasis on the reconstructive aspects of remembering. The real possibility exists that these deficiencies account for the fact that for so many gifted individuals' schooling becomes a dull routine from which one must escape from as rapidly as possible. The five R's of Remembering (and therefore learning) are: representation, rehearsal, reconstruction, registration, and rearrangement.

These four processes can be compared with the three major processes that were identified by Piaget as responsible for cognitive growth: evaluation, accommodation, assimilation and finally equilibration.

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